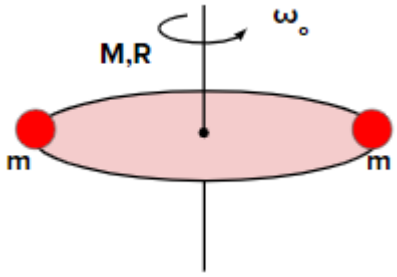
JEE-Main-18-03-2021-Shift-1 (Memory Based)

PHYSICS

Question: A disc of mass M and radius R is rotating about its axis with initial angular velocity of ω_0, as shown. Now two small masses of m each, kept on the circumference diametrically opposite to each other.
Find the new angular velocity.



Options:

(a) $\frac{M\omega_0}{M+2m}$

(b) $\frac{M\omega_0}{M+m}$

(c) $\frac{M\omega_0}{M+4m}$

(d) None

Answer: (c)

Solution:

As no external torque is there, So Angular momentum will remain conserved:

$$\left|\vec{L}_i\right| = \left|\vec{L}_f\right|$$

$$\Rightarrow \frac{1}{2}MR^2\omega_0 = \left(\frac{1}{2}MR^2 + 2mR^2\right)\omega$$

$$\Rightarrow \frac{1}{2}MR^2\omega_0 = \frac{(M+4m)R^2}{2}\omega$$

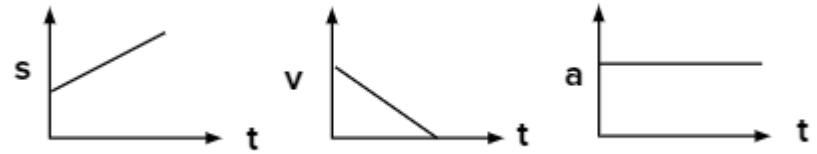
$$\Rightarrow \omega = \frac{M}{(M+4m)}\omega_0$$

Question: 4 sets of graphs one given. Each set consists of a displacement - time (s - t), velocity time (v - t) & acceleration time (a - t) graph. Which set correctly illustrates all 3 graphs.

Options:

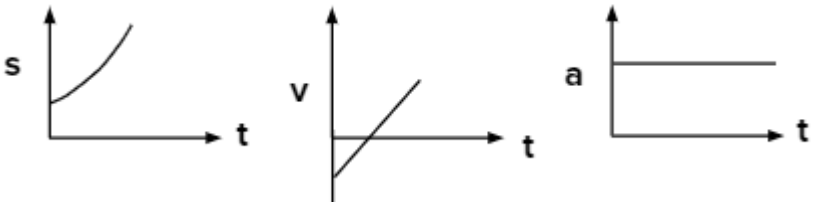
(a)

(b)



(c)

(d)



Answer: (a)

Solution:

If we consider:

$x(t) \propto t^2$

So, position – time curve would be parabola

$\Rightarrow$ Velocity: $v(t) = \frac{d}{dt} x(t)$

$v(t) \propto t^1$

So, velocity time curve would be straight line (with slope $\neq 0$)

$\Rightarrow$ Acceleration: $a(t) = \frac{d}{dt} v(t)$

$a(t) \propto t^0$

So, acceleration would be constant and acceleration time curve would be parallel to time axis.
So, only option (A) satisfies, all the conditions.

Question: A bullet of mass 0.1 kg goes inside a block with an initial speed of 10 m/s. It travels 50 cm inside the block and stops. Find the magnitude of retarding force if we assume its magnitude to remain constant:

Options:
(a) 1 N
(b) 10 N
(c) 100 N
(d) 50 N
Answer: (b)
Solution:

$$v^2 - u^2 = 2\vec{a}.\vec{S}$$

$$\Rightarrow (0)^2 - (+10)^2 = 2(\vec{a})\left(+\frac{50}{100}\right)$$

$$\Rightarrow \vec{a} = -100\, m/s^2$$

($-ve$ sign show acceleration is in opposite direction of velocity).

$$\Rightarrow |\vec{F}| = m|\vec{a}|$$

$$\Rightarrow |\vec{F}| = (0.1)(100) = 10\, N$$

Question: In series LCR circuit if the resistance is increased keeping L and C same for the same voltage AC source. Then which is true?
Options:
(a) Quality factor and bandwidth frequency increases
(b) Quality factor and bandwidth frequency won't change
(c) Quality factor decreases, bandwidth frequency increases
(d) Quality factor increase, bandwidth frequency won't change
Answer: (a)
Solution:

Quality factor $Q = \frac{X_L}{R}$ $\quad X_L = 2\pi fL$

$$Q = \frac{2\pi fL}{R}$$

With increase in resistance quality factor will decrease.

Band width $(BW) = \frac{f}{Q}$

$$BW = \frac{f}{\frac{2\pi fL}{R}}$$

$$BW = \frac{R}{2\pi L}$$

With increase in resistance band width (BW) will increases.

Question: A radioactive material X decays via two processes to produce Y & Z parallelly with half lives 1 hr and 2 hr respectively. Find the effective half life.

Options:

(a) 3 hrs

(b) $\frac{3}{2} hrs$

(c) $\frac{2}{3} hrs$

(d) 1 hrs

Answer: (c)

Solution:

Equation for parallel disintegration can be written as $-\frac{dN_x}{dt} = \lambda_1 N_x + \lambda_2 N_x$

$$\Rightarrow -\frac{dN_x}{dt} = (\lambda_1 + \lambda_2) N_x$$

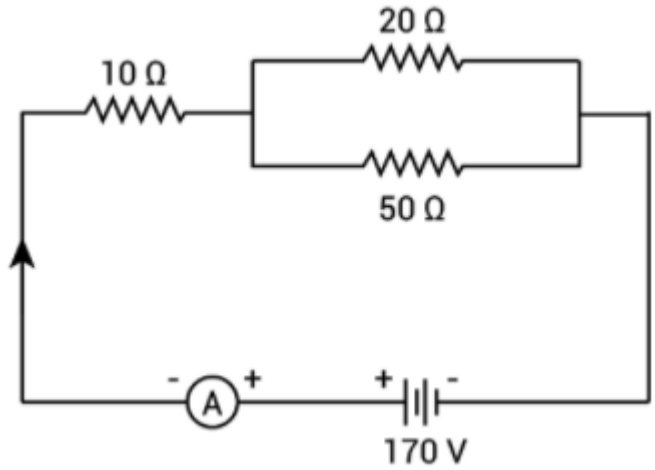
Effective decay in parallel disintegration is given by $\lambda_P = \lambda_1 + \lambda_2$

$$\lambda_X = \lambda_Y + \lambda_Z$$

$$\frac{0.693}{T_X} = \frac{0.693}{T_Y} + \frac{0.693}{T_Z}$$

$$T_X = \frac{2}{3} hr$$

Question: In the given circuit find the potential difference across 10Ω resistance.



Options:

(a) 100 V

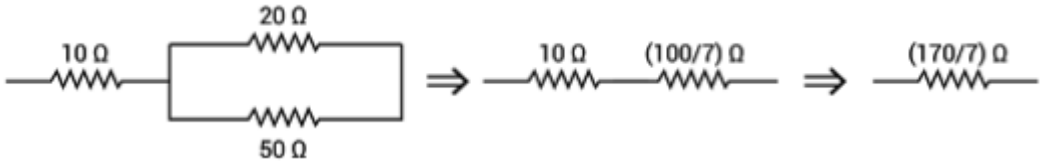
(b) 10 V

(c) 70 V

(d) None of these

Answer: (c)

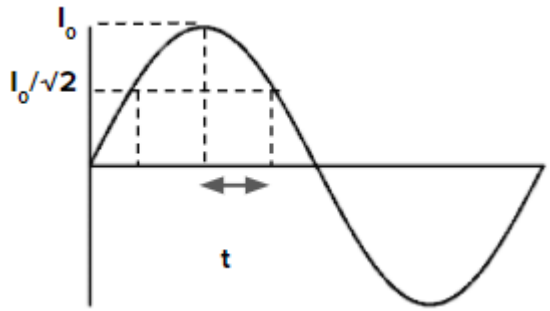
Solution:



$$I = \frac{E}{R_{eff}} = \frac{170}{(170/7)}$$

$$V_{10\Omega} = I \times 10 = \frac{170}{(170/7)} \times 10 = 70V$$

Question: In an AC source of 220v and 50Hz, the time taken for the current to go from peak value to RMS is



Options:

(a) 0.25 ms

(b) 0.25 sec

(c) 2.5 ms

(d) 2.5 sec

Answer: (c)

Solution:

$$I = I_0 \sin \omega t$$

$$\frac{I_0}{\sqrt{2}} = I_0 \sin 2\pi vt$$

$$2\pi vt = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{1}{8 \times 50} = 2.5 \times 10^{-3} s$$

Question: Velocity of particle = 4 time velocity of electron if $\frac{\lambda_{\text{particle}}}{\lambda_e} = \frac{2}{1}$, Find $\frac{m_p}{m_e} =$

Options:

(a) 8

(b) 1/8

(c) 4

(d) 1/4

Answer: (b)

Solution:

Given

$$\frac{V_{\text{particle}}}{V_e} = \frac{4}{1}$$

$$\frac{\lambda_{\text{particle}}}{\lambda_e} = \frac{2}{1}$$

$$P = \frac{hc}{\lambda}$$

$$mv = \frac{hc}{\lambda}$$

$$m_P V_P = \frac{hc}{\lambda_P} \quad ...(i)$$

$$m_e V_e = \frac{hc}{\lambda_e} \quad ...(ii)$$

Dividing equation (i) and (ii)

$$\frac{m_P V_P}{m_e V_e} = \frac{\lambda_e}{\lambda_P}$$

$$\frac{m_P}{m_e} = \frac{\lambda_e}{\lambda_P} \cdot \frac{V_e}{V_P}$$

$$\frac{m_P}{m_e} = \frac{1}{2} \times \frac{1}{4}$$

$$\frac{m_P}{m_e} = \frac{1}{8}$$

Question: In a YDSE experiment the wavelength of light used is $\lambda = 5890\,\mathring{A}$. The separation between the slits is d = 0.5 mm, and the distance between the slits and the screen is D = 0.5 m. Find the distance between 1st and 3rd bright fringes on the screen?

Options:

(a) $589\,\mu m$

(b) $1767\,\mu m$

(c) $1178\,\mu m$

(d) None of these

Answer: (c)

Solution:

$$y_n = \frac{n\lambda D}{d}$$

$$\lambda = 5890\,\mathring{A} = 5890 \times 10^{-10}\,m$$

$$D = 0.5\,m$$

$$d = 0.5\,mm = 0.5 \times 10^{-3}\,m$$

$$y_1 = \frac{1 \times \lambda D}{d}$$

$$y_3 = \frac{3\lambda D}{d}$$

Distance between 1st and 3rd bright fringe is

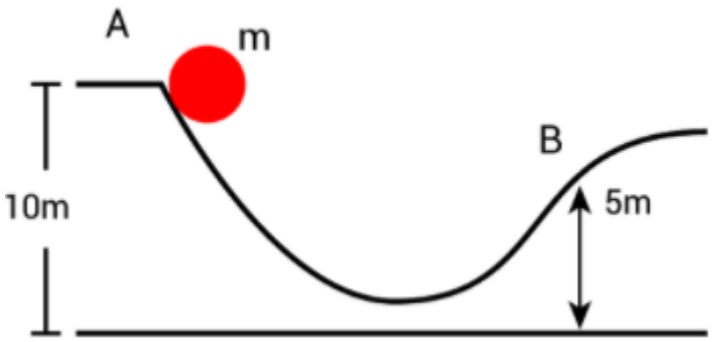
$$y_3 - y_1 = \frac{3\lambda D}{d} - \frac{\lambda D}{d}$$

$$y_3 - y_1 = \frac{2\lambda D}{d}$$

$$y_3 - y_1 = \frac{2 \times 5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$$

$$y_3 - y_1 = 1178\,\mu m$$

Question: A body of mass 10kg, is at a height of 10m at point A as shown on a smooth track. Find the speed at B?



Options:

(a) 5 m/s

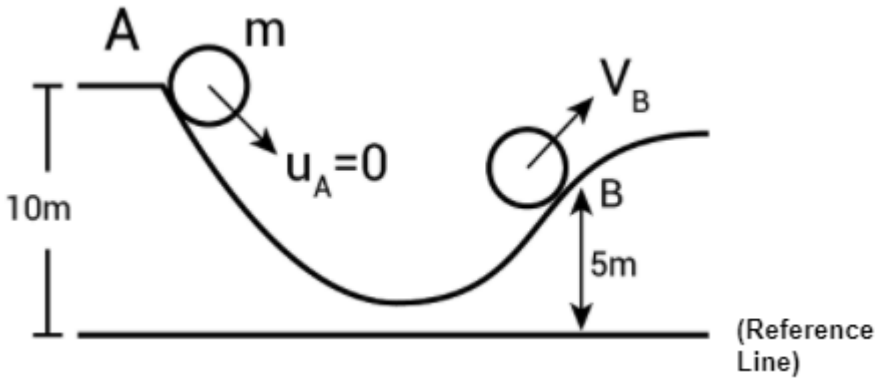
(b) $\sqrt{20}m/s$

(c) $10\,m/s$

(d) $20\,m/s$

Answer: (c)

Solution:



Applying conservation of energy

$$(P.E)_A+(K.E)_A=(P.E)_B+(K.E)_B$$

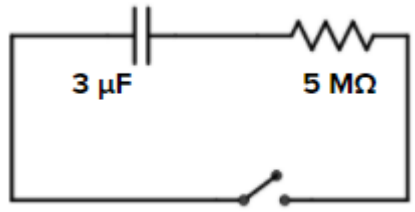
$$mg\times 10+0=mg\times 5+\frac{1}{2}mv_B^2$$

$$10mg=5mg+\frac{1}{2}mv_B^2$$

$$5g=\frac{1}{2}v_B^2$$

$$v_B=10\,m/s$$

Question: Initial charge on capacitor of $3\,\mu F$ is $30\,\mu C.$ Find the initial current when switch is closed?



Options:

(a) $1\mu A$

(b) $2\mu A$

(c) $3\mu A$

(d) $4\mu A$

Answer: (b)

Solution:

Charging of capacitor is given by

$$Q(t)=Q_0\left(1-e^{-t/Rc}\right)$$

Differentiating both side w.r.t time

$$I(t)=\frac{Q_0}{RC}e^{-t/RC}$$

At $t=0$

$$I(0)=\frac{Q_0}{RC}$$

Given $Q_0=30\mu C$

$R=5M\Omega$

$C=3\mu F$

$RC=(3\times5)s=15s$

$$I(0)=I_0=\frac{30\mu C}{15s}=2\mu A$$

Question: An electromagnetic wave of 100 MHz is travelling in +X direction and the magnetic field at origin at an instant is $2\times10^{-8}T\ \hat{k}$. Find $\vec{E}(inV/m)$ at origin at same instant?

Options:

(a) $0.6\hat{j}$

(b) $6\hat{j}$

(c) $0.6k$

(d) $6\hat{k}$

Answer: (b)

Solution:

$\Rightarrow f = 100\,MHz$

We know that

$|E| = c|B|$

As wave is going in X-direction and magnetic field is along Z-direction so from

$\vec{S} = \vec{E} \times \vec{B}$

We get direction of E along Y-direction

$|E| = 3 \times 10^8 \times 2 \times 10^{-8}$

$E = 6V/m$

$\vec{E} = (6V/m)\,j$

Question: In a hydrogen atom if electron is replaced by a muon, then find the Ionisation energy of the atom? (Given : $m_\mu = 207 m_e$)

Options:

(a) 13.6 eV

(b) 27.2 eV

(c) 2530 eV

(d) 2720 eV

Answer: (c)

Solution:

We know that, for hydrogen atom

$$E = (R_H)_\infty hc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad ....(i)$$

$(R_H)_\infty$ is the Rydberg constant when nucleas assume to be much heavier and it is at rest

In that case $(R_H)_\infty = \frac{m_e e^4}{8\varepsilon^2 h^3 c}$,similiarly $R_\mu = 207(R_H)_\infty$

When we replace electron with muon and muon is 207 times heavier than electron

So,

We have

$$R_H = \frac{(R_H)_\infty}{1+\frac{m_e}{M_H}}$$ (When nucleus is not at rest but orbiting COM)

$$R_\mu = \frac{207(R_H)_\infty}{1+\frac{207}{1840}} = \frac{207}{1.1125}(R_H)_\infty$$

$$R_\mu = 186.06(R_H)_\infty$$

From eq (1)

Ionization energy of hydrogen atom

$n_1 = 1$ & $n_2 = \infty$

$$E = (R_H)_\infty hc\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) = -13.6eV \quad ....(i)$$

Ionization energy of muonic Hydrogen atom is E_μ

Similarly,

$$E_\mu = R_\mu hc\left(\frac{1}{n_1^2}-\frac{1}{n_f^2}\right)$$

$$= 186.06(R_H)_\infty hc\left(\frac{1}{n_1^2}-\frac{1}{n_f^2}\right)$$

$$= 186.06\times 13.6$$

$$= 2530.5 \simeq 2530eV$$

Question: A satellite is moving in a radius of R with time period T. Find the time period if the radius of the satellite is made 9R:

Options:

(a) 3 T

(b) a T

(c) 27 T

(d) T

Answer: (c)

Solution:

Using Kepler's third law

$T^2 \propto R^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\left(\frac{T}{T_2}\right)^2 = \left(\frac{R}{9R}\right)^3$$

$$\frac{T_2^2}{T^2} = (729)$$

$$T_2 = 27T$$

Question: Your friend has an eyesight problem in which he can't see a distant uniform window much clearly. It appears to him distorted and non uniform. What is he suffering from?

Options:

(a) Myopia & Astigmatism

(b) Presbyopia & Astigmatism

(c) Myopia & Hypermetropia

(d) Astigmatism

Answer: (a)

Solution:

Near-sightedness (myopia) is a common vision condition in which you can see objects near to you clearly, but objects farther away are blurry.

Astigmatism is a type of refractive error in which the eye does not focus light evenly on the retina. This results in distorted or blurred vision at any distance.

So, he is suffering from both eye defects, astigmatism, and myopia.

JEE-Main-18-03-2021-Shift-1 (Memory Based)

CHEMISTRY

Question: Match the following.

Column I	**Column II**
(A) Troposphere	**i)** Above 80 km
(B) Stratosphere	**ii)** 80 km
(C) Mesosphere	**iii)** 50 km
(D) Thermosphere	**iv)** 10 km

Options:

(a) A $\rightarrow$ (iv); B $\rightarrow$ (iii); C $\rightarrow$ (ii); D $\rightarrow$ (i)

(b) A $\rightarrow$ (ii); B $\rightarrow$ (iii); C $\rightarrow$ (iv); D $\rightarrow$ (i)

(c) A $\rightarrow$ (i); B $\rightarrow$ (iii); C $\rightarrow$ (iv); D $\rightarrow$ (ii)

(d) A $\rightarrow$ (iii); B $\rightarrow$ (ii); C $\rightarrow$ (i); D $\rightarrow$ (iv)

Answer: (a)

Solution:

Troposphere $\Rightarrow$ 10 km

Stratosphere $\Rightarrow$ 50 km

Mesosphere $\Rightarrow$ 80 km

Thermosphere $\Rightarrow$ Above 80km

Question: Match the following.

Ores (Column I)	**Chemical formula (Column II)**
(A) Hematite	**I)** Fe_3O_4
(B) Magnetite	**II)** $Al_2O_3.2H_2O$
(C) Bauxite	**III)** $CuCO_3.Cu(OH)_2$
(D) Malachite	**IV)** Fe_2O_3

Options:

(a) A → (IV); B → (I); C → (II); D → (III)

(b) A → (III); B → (II); C → (I); D → (IV)

(c) A → (II); B → (III); C → (IV); D → (I)

(d) A → (I); B → (IV); C → (III); D → (II)

Answer: (a)

Solution:

Hematite ⇒ Fe_2O_3

Magnetite ⇒ Fe_3O_4

Bauxite ⇒ $Al_2O_3.2H_2O$

Malachite ⇒ $CuCO_3.Cu(OH)_2$

Question: S1: Removal of hardness of water involves decomposition of $Mg(HCO_3)_2$ in $MgCO_3$.

S2: K_{sp} of $Mg(OH)_2$ > K_{sp} of $MgCO_3$

Options:

(a) Both S1 and S2 are correct

(b) S1 is correct, S2 is incorrect

(c) S2 is correct, S1 is incorrect

(d) Both S1 and S2 are incorrect

Answer: (a)

Solution: Hardness is usually caused due to thermally unstable magnesium bicarbonate and calcium bicarbonate. Magnesium bicarbonate decomposes into magnesium carbonate, carbon dioxide and water

Hence, S1 is correct

2) a) $Mg(OH)_2$

$K_{sp} = (1)^2.\ (2)^2.\ S^{2+1} = 4s^3$

$MgCO_3$

$K_{sp} = s^2$

Question: For which of the following orbitals, (Given, $l = 0$), the number of radial nodes is equal to 2.

Options:

(a) 2p

(b) 3s

(c) 2s

(d) 3p

Answer: (b)

Solution: $l = 0$ means s-subshell

Radical node $= n - l - 1$

$= 3 - 0 - 1 = 2$

Question: Ethane $\xrightarrow{O_2} CO_2 + H_2O$

Given that moles of ethane is 0.1 and the number of molecules of $H_2O = X \times 10^{22}$. Find X

Options:

(a) 18

(b) 9

(c) 28

(d) None of these

Answer: (a)

Solution:

$$C_2H_6 + \frac{7}{2}O_2 \rightarrow 2CO_2 + 3H_2O$$

0.1 moles $X \times 10^{22}$ molecule

1 mole (3 moles)

1 mole ethane on combustion produce 3 moles of H_2O

0.1 mole ethane will produce 0.3 mole of H_2O

1 mole of H_2O has 6.022×10^{23} molecules

0.3 moles of H_2O will have $\frac{3}{10} \times 6.022 \times 10^{23}$

$= 18.066 \times 10^{22}$ molecule

$X \times 10^{22} = 18.066 \times 10^{22}$

$X = 18.066 \approx 18$

Question: Which of the following on hydrolysis gives sugar
Options:

(a) Sucrose

(b) Glucose

(c) Fructose

(d) None of these

Answer: (a)

Solution: Sucrose is a disaccharide, on hydrolysis it gives glucose and fructose

Question: A in HCP and M occupies ⅔ tetrahedral voids. Find formula of the compound.
Options:

(a) M_2A_3

(b) M_4A_3

(c) M_2A_4

(d) M_8A_7

Answer: (b)

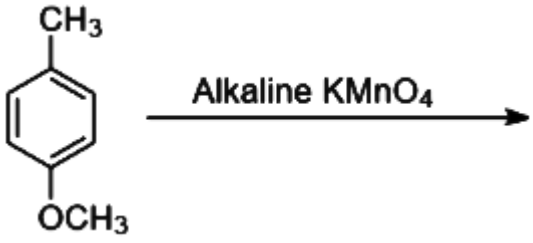
Solution: Number of atoms of A = 6

Number of atoms of M = $\frac{2}{3} \times 12$ (tetrahedral voids) $\Rightarrow 8$

M_8A_6

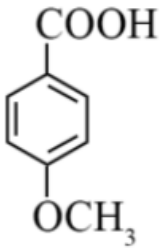
M_4A_3

Question:

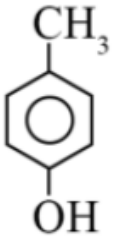


Options:

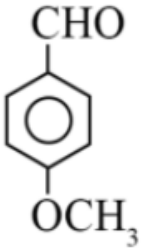
(a)



(b)



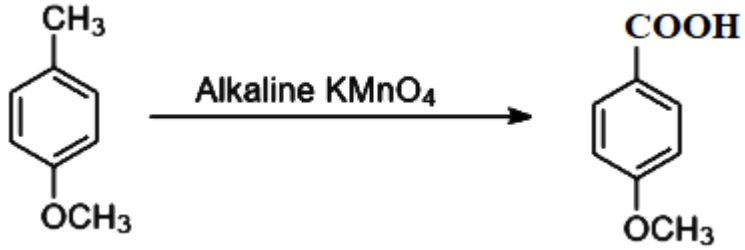
(c)



(d)

Answer: (a)

Solution:



Question: Match the following.

(Column I)	**Chemical formula (Column II)**

(A) CaO	**I)** Cement
(B) $CaCO_3$	**II)** Antacid
(C) $CaSO_4 . ½ H_2O$	**III)** Plaster of paris
(D) $CaOCl_2$	**IV)** Bleach

Options:

(a) A → (i); B → (ii); C → (iii); D → (iv)

(b) A → (ii); B → (iii); C → (iv); D → (i)

(c) A → (i); B → (iii); C → (iv); D → (ii)

(d) A → (iii); B → (ii); C → (i); D → (iv)

Answer: (a)

Solution:

$CaO \Rightarrow$ Cement

$CaCO_3 \Rightarrow$ Antacid

$CaSO_4 . ½ H_2O \Rightarrow$ Plaster of paris

$CaOCl_2 \Rightarrow$ Bleach

Question: Radius of Na^+ is 1.07 Å, then radii of Mg^{2+} and Al^{3+} respectively:

Options:

(a) 0.98 Å, 0.90 Å

(b) 0.90 Å, 0.98 Å

(c) 1.15 Å, 1.25 Å

(d) 1.25 Å, 1.15 Å

Answer: (a)

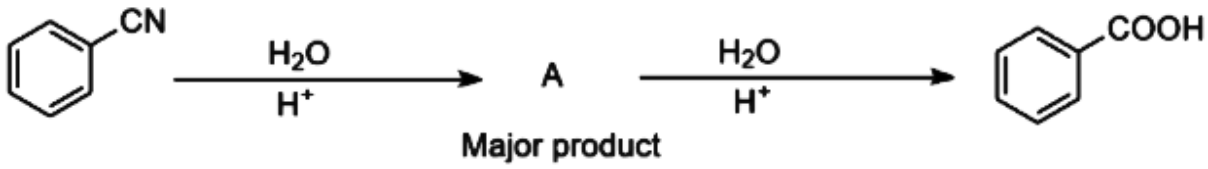
Solution:

As Mg^{+2} and Al^{+3} are isoelectronic, in case of isoelectronic species

$$\text{Atomic number } \alpha \frac{1}{\text{Atomic size}}$$

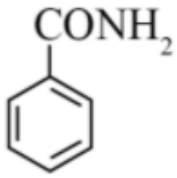
$\therefore$ Option (a) is correct

Question:

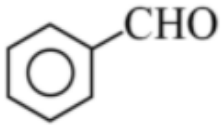


Options:

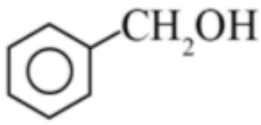
(a)



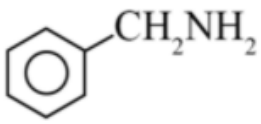
(b)



(c)

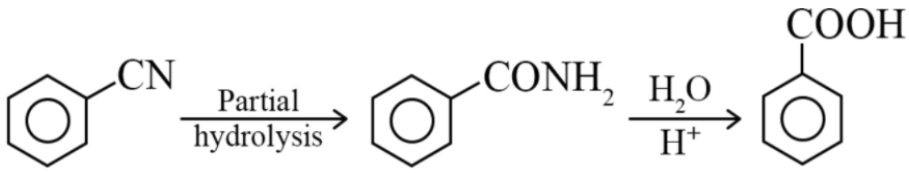


(d)



Answer: (a)

Solution:



Question: An electron of hydrogen is replaced by a particle of mass 217 times and having same charge. Find the energy to ionise it.

Options:

(a) 2.84×10^5 kJ/mole

(b) 1.8×10^5 kJ/mole

(c) 3.84×10^5 kJ/mole

(d) 5.84×10^5 kJ/mole

Answer: (a)

Solution:

$$E_x = \frac{-2\pi^2 me^4 z^2}{n^2 h^2} kJ / mole$$

Particle mass is 217 times mass of electron

$$E_n = 217 \times \left(-1312 \frac{z^2}{n^2} \right)$$

$z = 1, n = 1$

$E_n = 217 \times (-1312) = -284704$ kJ/mole

$= -2.84 \times 105$ kJ/mole

I.E. $= 0 - (-2.84 \times 10^5) = 2.84 \times 10^5$ kJ/mol

Question: HA (2 molal) has freezing point of -3.885°C. Find the degree of dissociation if $K_f = 1.85$

Options:

(a) 0.05

(b) 0.03

(c) 0.01

(d) 0.1

Answer: (a)

Solution:

$T_f = -3.885°C$

$\Delta T_f = 3.885$

$\Delta T_f = i \times K_f \times m$

$3.885 = i \times 1.85 \times 2$

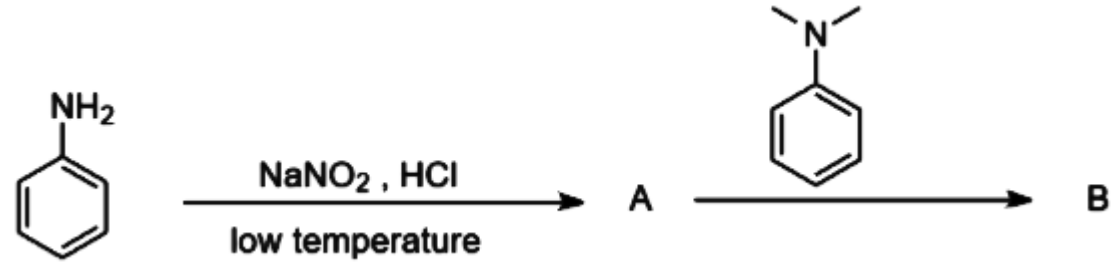
$i = 1.02$

$i = 1 + (n - 1)\alpha$

$1.05 = 1 + (-1)\alpha$

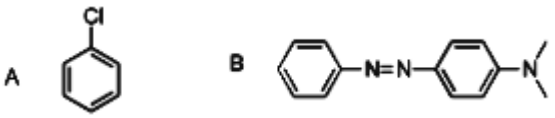
$0.05 = \alpha$

Question:

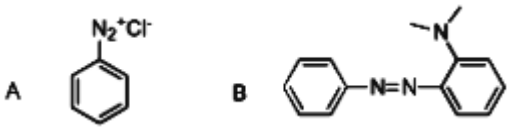


Options:

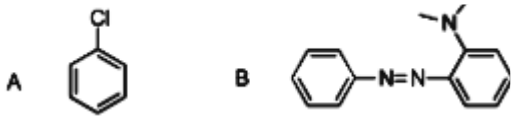
(a)



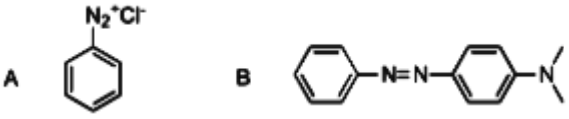
(b)



(c)

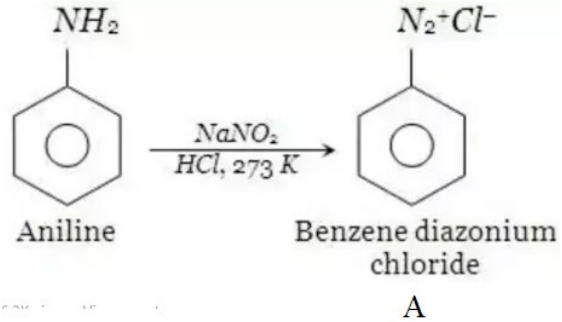


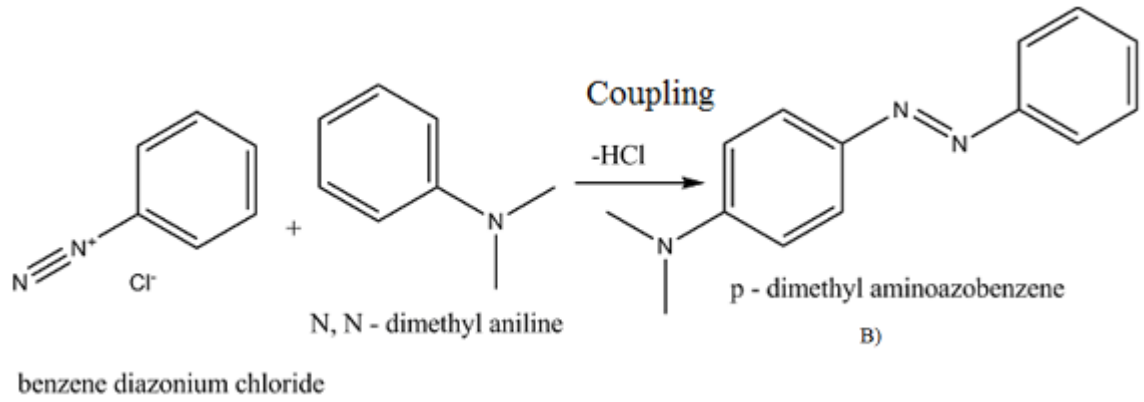
(d)



Answer: (d)

Solution:





Question: Number of unpaired electrons in $K_3[Cr(oxalate)_3]$ =?

Options:

(a) 3

(b) 4

(c) 2

(d) 1

Answer: (a)

Solution:

O.S of Cr = +3 ie d^3

Number of unpaired electrons = 3

Question: What is the reaction enthalpy of
$C_2H_6 \longrightarrow C_2H_4 + H_2$ and
B.E of C-C = 336.81 kJ/mol
B.E of C-H = 410.87 kJ/mol
B.E of H-H = 431.79 kJ/mol
B.E OF C=C = 606.68 kJ/mol

Options:

(a) –120.08 kJ/mol

(b) +120.08 kJ/mol

(c) –240.16 kJ/mol

(d) +240.16 kJ/mol

Answer: (b)
Solution:
$H(C_2H_4) = H(C=C) + 4 \times H\ (C–H)$
$= 606.68 + 4 \times 410.87 = 2250.16 kJ/mol$
Similarly,
$H(H_2) = 431.79$ kJ/mol
and
$H(C_2H_6) = H(C–C) + 6 \times H(C–H) = 336.81 + 6 \times 410.87 = 2802.03$ kJ/mol
$\therefore H_{(reaction)} = H(C_2H_6) - [H(H_2) + H(C_2H_4)]$
$H_{(reaction)} = 2802.03 - (431.79 + 2250.16) = 120.08$ kJ/mol.

Question: A buffer has a pH 5.6, pK_a of acetic acid is 4.76, concentration of acid is 1 M. Find the concentration of salt.
Options:

(a) 7M

(b) 6M

(c) 2M

(d) 5M

Answer: (a)

Solution: $pH = pK_a + \log\frac{[salt]}{[Acid]}$

$5.6 = 4.76 + \log\frac{(x)}{(1)}$

$0.84 = \log x$

$x = 6.92 \approx 7$ M

Question: Gold(I) thiomalate is used as a modified for the treatment of which of the following

Options:

(a) Malaria

(b) Arthritis

(c) Diabetes

(d) Ulcer

Answer: (b)

Solution: It is used for treatment of Arthritis

Question: Diamagnetic materials are

Options:

(a) B_2, C_2, N_2

(b) O_2, N_2, F_2

(c) C_2, N_2, F_2

(d) B^2, O_2^{2-}, N_2

Answer: (d)

Solution: C_2, N_2, F_2 all are diamagnetic

Question: Which of the following is not a functional isomeric of C_3H_6O?

Options:

(a) Propanol

(b) Propanal

(c) Propanone

(d) Cyclopropanol

Answer: (a)

Solution: Propanol (C_3H_8O) is not a functional isomer of C_3H_6O, whereas propanal, Propanone and Cyclopropanol have molecular formula (C_3H_6O)

Question: The reaction of glucose with acetic anhydride and Tollen's reagent suggest that is

Options:

(a) Pentahydroxy aldehyde

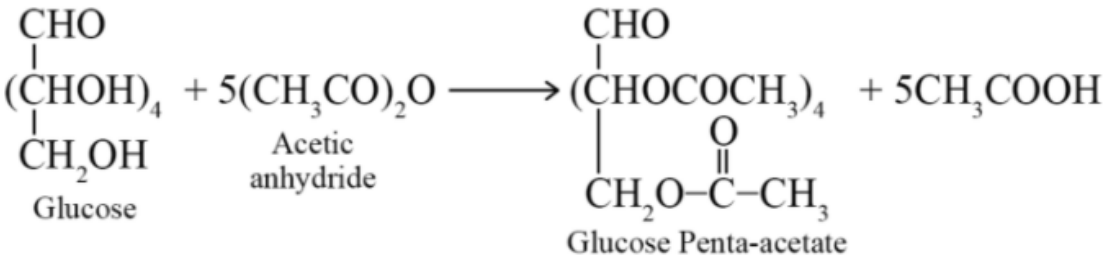
(b) Polyhydroxy ketone

(c) Glucose pentaacetate

(d) None of these

Answer: (c)

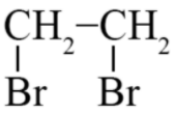
Solution:



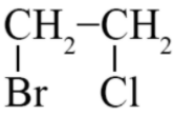
Question: Reaction of Br_2 on ethylene in presence of NaCl gives the compound as major product is

Options:

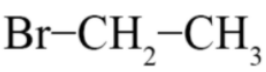
(a)



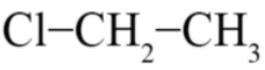
(b)



(c)



(d)



Answer: (b)

Solution:

JEE-Main-18-03-2021-Shift-1 (Memory Based)

MATHEMATICS

Question: If $f(x)=\frac{\text{cosec}^{-1}x}{\sqrt{x^2-[x]^2}}$, find deviation

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$$f(x)=\frac{\text{cosec}^{-1}x}{\sqrt{x^2-[x]^2}}$$

For $\sqrt{x^2-[x]^2}$

$x^2 \geq [x]^2$

$x \geq [x]$

$[x]+\{x\} \geq [x]$

$\{x \geq 0$

$\Rightarrow x \geq 0 \Rightarrow x \in [0,\infty)$

Here we have $\frac{1}{\sqrt{x^2-[x]^2}} \Rightarrow x \notin Z$

For $\text{cosec}^{-1}x$, $|x| \geq 1$

$\Rightarrow x \in (1,\infty)-Z$ Domain

Question: $f(x)=\sqrt{x}, g(x)=\sqrt{1-x}$, find common domain of $f+g, f-g, \frac{f}{g}, \frac{9}{f}$

Options:

(a) $x \in (0,1)$

(b) $x \in [0,1)$

(c) $x \in [0,1]$

(d) $x \in (0,1]$

Answer: (a)

Solution:

For $f+g,\ f-g$

$D_f \cap D_g$

$$\left.\begin{array}{ll} \dfrac{f}{g}, D_f \cap D_g,\ g \neq 0 & [0,1] \cap x \neq 1 \\ \dfrac{g}{f},\ D_f \cap D_g,\ f \neq 0 & [0,1] \cap x \neq 0 \end{array}\right\} (0,1) \quad \ldots(1)$$

$D_f : x \geq 0$

$D_g : 1-x \geq 0 \Rightarrow x \leq 1$

$D_f \cap D_g = [0,1] \quad \ldots(2)$

Common domain is (A) $x \in (0,1)$

Question: Find differentiated equation of $y^2 = 4a(x+a)$

Options:

(a) $y\left(\dfrac{dy}{dx}\right)^2 + 2x\dfrac{dy}{dx} - y = 0$

(b) $y\left(\dfrac{dy}{dx}\right)^2 - 2x\dfrac{dy}{dx} - y = 0$

(c)

(d)

Answer: (a)

Solution:

$y^2 = 4ax + 4a^2$

$2yy' = 4a$

$\Rightarrow a = \dfrac{yy'}{2}$

We have $y^2 = 4a(x+a)$

$$y^2 = 4 \cdot \frac{yy'}{2}\left(x + \frac{yy'}{2}\right)$$

$$y^2 = 2xy\frac{dy}{dx} + 2yy' \cdot \frac{yy'}{2}$$

$$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow y = 2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2$$

Question: If $\left(1+x+2x^2\right)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$ then $a_1 + a_3 + a_5 + ... + a_{39} =$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

. Put $x = 1$

$$4^{20} = a_0 + a_1 + a_2 + ... + a_{40} \quad ....(1)$$

$$x = -1$$

$$\left(1-1+2\right)^{20} = a_0 - a_1 + a_2 - a_3 .... a_{40} \quad ...(2)$$

$$(1)-(2)$$

$$4^{20} - 2^{20} = 2\left(a_1 + a_3 + ... + a_{39}\right)$$

$$\Rightarrow a_1 + a_3 + ... + a_{39} = \frac{2^{20}\left(2^{20}-1\right)}{2}$$

$$= 2^{19}\left(2^{20}-1\right)$$

Question: $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + ... + \frac{1}{201^2-1} =$ is equal to

Options:

(a)

(b)
(c)
(d)

Answer: $\frac{25}{101}$

Solution:

$$T_n = \frac{1}{(2n+1)^2} = \frac{1}{(2n+2)(2n)}$$

$$= \frac{1}{2}\left(\frac{2n+2-2n}{(2n+2)(2n)}\right)$$

$$= \frac{1}{2}\left(\frac{1}{2n} - \frac{1}{2n+2}\right)$$

$$= \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n+2}\right)$$

$$= \frac{1}{4}\left(1 - \frac{1}{101}\right)$$

$$= \frac{25}{101}$$

Question: The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\ldots\infty}}}}$ is equal to

Options:

(a) $4+\sqrt{3}$

(b) $1.5+\sqrt{3}$

(c) $2+\sqrt{3}$

(d) $3+2\sqrt{3}$

Answer: (b)

Solution:

$$x = 3+\frac{1}{4+\frac{1}{x}}$$

$$x = 3+\frac{x}{4x+1}$$

$$(4x+1)x = 12x+3+x$$

$$4x^2 + x - 12x - 3 - x = 0$$

$$4x^2 - 12x - 3 = 0$$

$$x = \frac{12 \pm \sqrt{144 + 4 \cdot 4 \cdot 3}}{2 \cdot 4} = \frac{12}{8} \pm \frac{8\sqrt{3}}{8}$$

$$= 1.5 + \sqrt{3}$$

Question: M, N, O, P are 4 circles $x^2 + y^2 = 1, x^2 + y^2 - 2x = 0, x^2 + y^2 - 2y = 0, x^2 + y^2 - 2x - 2y = O.$ Centers of these circles are joined then shape formed is

Options:

(a) Rhombus

(b) Rectangle

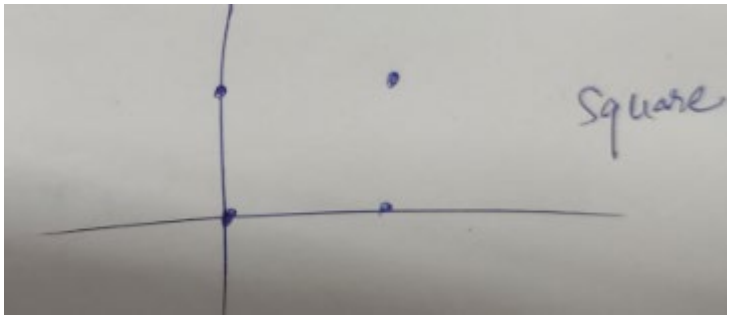
(c) Square

(d) Parallelogram

Answer: (c)

Solution:

		Center
M:	$(x-0)^2 + (y-0)^2 = 1$	$(0,0)$
N:	$(x-1)^2 + y^2 = 1$	$(1,0)$
O:	$x^2 + (y-1)^2 = 1$	$(0,1)$
P:	$(x-1)^2 + (y-1)^2 = 1$	$(1,1)$



Question: $\bar{a}$ vector has components $3P$ and 1 in rectangular Cartesian system. If $\bar{a}$ is rotated counterclockwise abut origin such that its components now becomes $\sqrt{10}$ and $P+1$. Then a values of 'P' is

Options:

(a) $\frac{-5}{4}$

(b) $\frac{4}{5}$

(c) 1

(d) -1

Answer: (d)

Solution:

. Only rotated so magnitude remains same:

$$\Rightarrow \sqrt{9P^2+1} = \sqrt{\left(\sqrt{10}\right)^2 + \left(P+1\right)^2}$$

$$\Rightarrow 9P^2+1 = 10 + P^2 + 1 + 2P$$

$$8P^2 - 2P - 10 = 0$$

$$4P - P - 5 = 0$$

$$P = \frac{-4}{4}, \frac{5}{4}$$

$$= -1, \frac{5}{4}$$

Question: $f\left(x\right) = \begin{cases} \frac{1}{|x|} & ; |x| \geq 1 \\ ax^2 + b & ; |x| < 1 \end{cases}$ Find possible of a and b ? Given that $f\left(x\right)$ is differentiable

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

. $f\left(x\right) = \begin{cases} \frac{1}{x} & ; \quad x \geq 1 \\ ax^2 + b & ; \quad x < 1 \\ ax^2 + b & ; \quad x > -1 \\ \frac{-1}{x} & ; \quad x \leq -1 \end{cases}$

Continuous at $x = 1, -1,\ a + b = 1 \qquad ....(1)$

$$f'(x)=\begin{cases}\dfrac{-1}{x^2} & ; \quad x\geq 1\\ 2ax & ; \quad x<1\\ 2ax & ; \quad x>-1\\ \dfrac{1}{x^2} & ; \quad x\leq -1\end{cases}$$

$x=\pm 1,\ 2a=-1 \qquad ....(2)$

$2a+2b=2 \Rightarrow a=\dfrac{-1}{2}$

$2b=2+1=3$

$b=\dfrac{3}{2}$

Question: If $I=\int \dfrac{(2x-1)\cos\sqrt{4x^2-4x+6}}{\sqrt{4x^2-4x+6}}\,dx$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$(2x-1)^2+5=4x^2-4x+6$

Let $(2x-1)^2+5=t^2$

$2(2x-1)\cdot 2\,dx=2t\,dt$

$I=\int t\dfrac{dt}{2}\cdot\dfrac{\cos t}{t}=\dfrac{1}{2}\int \cos t\,dt$

$=\dfrac{1}{2}\sin t+c$

$=\dfrac{1}{2}\sin\sqrt{4x^2-4x+6}$

Question: $x^2+y^2-10x-10y+41=0$ and $x^2+y^2-22x-10y+137=0$

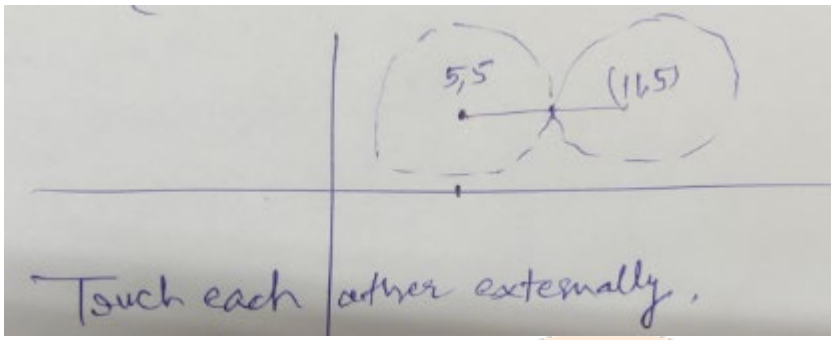
Options:

(a) Meet at 1 point

(b) Meet at 2 points
(c) Does not meet
(d) Have same center
Answer: (a)
Solution:

$$(x-5)^2+(y-5)^2=3^2$$

$$(x-11)^2+(y-5)^2=3^2$$



Question: $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$ solutions in $x \in (0,\pi)$

Options:
(a)
(b)
(c)
(d)
Answer: ()
Solution:

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 2+4\sin 2x & 2+4\sin 2x & 2+4\sin 2x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix}$$

$$= 2+4\sin 2x \begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix}$$

$$= |2+4\sin 2x| \begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & \cos^2 x \\ 1 & -1 & 1+4\sin 2x \end{vmatrix}$$

$\Rightarrow 2+4\sin 2x=0,\ x\in(0,\pi)$

$\sin 2x=\frac{-1}{2},\ 2x\in(0,2\pi)$

$x=\frac{7\pi}{12},\frac{11\pi}{12}$

Question: The sum of all the 4- digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is :

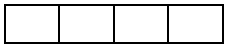
Answer: 26664.00

Solution:

1, 2, 2, 3

For one's place

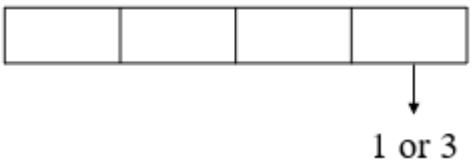
Case 1:



Total numbers are $3!=6$

$\therefore$ Sum of numbers at one's place = $(2)\times1\times6=12$

Case 2:



Total numbers are $\frac{3!}{2!}=3$

$(1+3)\times1\times3=12$

Total = 12 + 12 = 24

Similarly,

$(2\times10\times6)+(1+3)\times10\times3=240$

$(2\times100\times6)+(1+3)\times100\times3=2400$

$(2\times1000\times6)+(1+3)\times1000\times3=24000$

Total sum = 26664

Question: The number of integral values of m so that the abscissa as point of intersection of lines $3x+4y=9$ and $y=mx+1$ is also an integers is

Answer: 2.00

Solution:

Lines are:

$3x+4y=9$

$4mx-4y=-4$

$(4m+3)x=5$

$$x=\frac{5}{4m+3}$$

$x\in Z$

$\Rightarrow 4m+3=5,1,-1,-5$

$4m+3=-1$

$m=-1$

$4m+3=-5$

$m=-2$

Question: $\lim\limits_{x\to 0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}=L$ then find $(6L+1)$

Answer: 2.00

Solution:

$$\sin^{-1}x=x+\frac{x^3}{6}....$$

$$\tan^{-1}x=x-\frac{x^3}{3}$$

$$\Rightarrow \lim_{x\to 0}\frac{x+\frac{x^3}{6}...-\left(x-\frac{x^3}{3}...\right)}{3x^3}=\frac{\frac{x^3}{6}+\frac{2x^3}{6}}{3\times 3}=\frac{1}{6}=L$$

$\Rightarrow 6L=1$

$6L+1=2$

Question: Find the number of times 3 appeared in all the numbers from 1 to 1000.
Answer: 300.00
Solution:
3 at different places can occur as:

$3 \quad 10 \quad 10 = 100$ ways

$10 \quad 3 \quad 10 = 100$ ways

$10 \quad 10 \quad 3 = \underline{100 \text{ ways}}$

300 ways/times

Question: The planes parallel to the $x - 2y + 2z - 3 = 0$ and are at a unit distance from point $(1,2,3)$ is $ax + by + cz + d = 0$. If $(b - d) = k(c - a)$. Find positive values of k?
Answer: 4.00
Solution:
$x - 2y + 2z + k = 0$ be the parallel plane

$$\frac{|1 - 2 \times 2 + 2 \times 3 + k|}{\sqrt{1^2 + 2^2 + 2^2}} = 1$$

$$|3 + k| = 3$$

$3 + k = 3, \quad 3 + k = -3$
$k = 0, \qquad k = -6$

Planes are:

$x - 2y + 2z = 0$ & $x - 2y + 2z - 6 = 0$

$a = 1,\ b = -2,\ c = 2,\ d = 0$

$a = 1,\ b = -2,\ c = 2,\ d = -6$

$$k = \frac{b - d}{c - a} = \frac{-2}{1}$$

$$k = \frac{4}{1} = 4$$

Question: Number of solution of equation $\left|\cot x\right| = \cot x + \frac{1}{\sin x}$ in $\left[0, 2\pi\right]$

Answer: 1.00

Solution:

If $\cot x > 0$

Then $\frac{1}{\sin x} = 0$ (impossible)

Now if $\cot x < 0$

Then $-\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow \frac{2\cos x + 1}{\sin x} = 0 \Rightarrow \cos x = \frac{-1}{2}$

$\therefore x = 2n\pi \pm \frac{2\pi}{3}, n \in I$ and $0 \le x \le 2\pi$

As $\cot x < 0$

$\Rightarrow x = \frac{2\pi}{3}$

Question: $f\left(x\right) = \int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)} dx$, If $f\left(0\right) = 0$, $f\left(1\right) = \frac{1}{k}$. Then k is

Answer: 4.00

Solution:

$\int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)} dx$

$\int \frac{x^{14}\left(\frac{5}{x^6} + \frac{7}{x^8}\right)}{x^{14}\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)} dx$

$\frac{1}{x^5} + \frac{1}{x^7} + 2 = t$

$\Rightarrow -5x^{-6} - 7x^{-8} dx = dt$

$f\left(x\right) = \int \frac{-dt}{t^2} = \frac{1}{t} + c$

$$f(x)=\frac{1}{\frac{1}{x^5}+\frac{1}{x^7}+2}+c$$

$$f(x)=\frac{x^7}{x^2+1+2x^7}+c$$

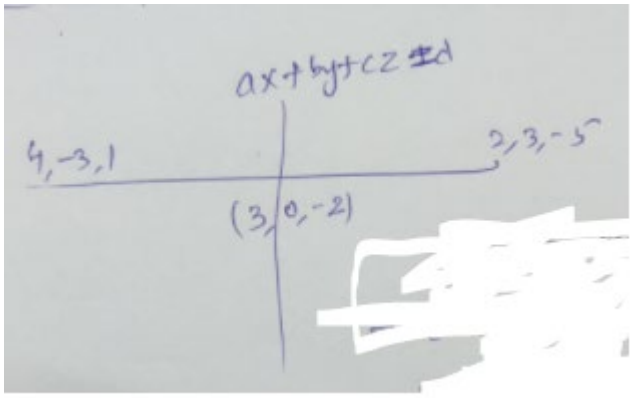
$$f(0)=0=0+c$$

$$c=0$$

Question: $ax+by+cz=d,$ If bisects the line joining $(4,-3,1)$ and $(2,3,-5)$ in perpendicular direction, Find minimum value of $a^2+b^2+c^2+d^2$, given that a,b,c,d are all integers.

Answer: 28.00

Solution:



$$\vec{n}=2\hat{i}-6\hat{j}+6\hat{k}$$

DR's is (1, -3, 3)

$$\Rightarrow x-3y+3z=d$$

Passes through (3, 0, -2)

$$3-6=d$$

$$d=3$$

$\Rightarrow$ Plane is $x-3y+3z=3$

$$a^2=1,\ b^2=c^2=d^2=9$$

$$\Rightarrow a^2+b^2+c^2+d^2=28$$

Question: There are 25 teachers in a school, the average age of teacher is 40. If a teacher of 60 Years of age is retired then a new teacher is appointed in place of him and the average decreases to 39, find the age of teacher appointed?

Answer: 35.00

Solution:

$n = 25$

$$\frac{\sum \text{ages}}{25} = 40$$

$$\frac{\sum \text{ages} - 60 + x}{25} = 39$$

$$25 \times 40 - 60 + x = 25 \times 39$$

$$x = 60 + (-25) = 35$$